

Fisher information manifestation of dynamical stability and transition to self-trapping for Bose-Einstein condensates

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We investigate dynamical stability and self-trapping for Bose-Einstein condensates in a symmetric double well. The relation between the quantum Fisher information and the stability of the fixed point is studied. We find that the quantum Fisher information displays a sharp transition as the fixed point evolving from stable to unstable regime. Moreover, the transition from Josephson oscillation to self-trapping is accompanied by an abrupt change of the quantum Fisher information.

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I. INTRODUCTION

Quantum Fisher information (QFI) which characterizes the sensitivity of the state with respect to changes of the parameter, is a key concept in parameter estimation theory [1, 2]. It has important applications in quantum technology such as quantum frequency standards [3, 4], measurement of gravity accelerations [5], and clock synchronization [6]. Recently, the QFI was found to be able to detect entanglement and quantum phase transitions in many-body systems [7–11]. In the Lipkin-Meskhan-Glick model, the QFI can be used to characterize the ground state which displays a second-order quantum phase transition [12]. The critical point of a transverse Ising chain can also be estimated by QFI [13, 14].

We use QFI to study the dynamical stability and the transition from Josephson oscillation (JO) to self-trapping (ST) for Bose-Einstein condensates (BECs) in a double well potential. This transition is an interesting finding in BECs [15–20]. By changing the atomic interaction, the Josephson oscillation may be blocked, and the atoms of a BEC in a symmetric double-well potential may show a highly asymmetric distribution between two wells. This phenomenon has been observed in the experiment [21].

Over the past few years, people found the transition is strongly related to the fixed points of the underlying classical dynamics. This is because quantum system can be mapped into a classical Hamiltonian which gives rise to fixed point solutions [22]. Recently, some researchers have explored that quantum entanglement which is closely related to QFI can manifest the transition [23–25]. People also found quantum entanglement has relation to the classical fixed-point bifurcation—a loss of stability and the emergence of new fixed points [26–29]. However, effects of the stabilities of the fixed points on quantum information flow is still lacking. It is thus certainly important to explore what features of the dynamical stabilities are manifested by the QFI.

We study the Hamiltonian of the system with the approach of spin coherent state (SCS) associated to SU(2) group. We localize all the fixed points of the model in the phase space and discuss their stabilities. We also analyze the transition from JO to ST regime. In particular, we focus on the dynamics of the QFI for different interaction regions, such as ST and JO, stable and unstable regimes. We find that QFI can clearly demonstrate the dynamical stability and the transition from JO to ST.

Our paper is organized as follows. In Sec. II, we discuss the parameter estimation, QFI, and introduce the maximal mean QFI. In Sec. III, we study the stabilities of the fixed points, and discuss the transition from JO to ST. Then in Sec. IV, we investigate the maximal mean QFI for two different initial states and reveal QFI manifestation of the stability of the fixed point and the transition. Finally, a summary is provided in Sec. V.

II. QUANTUM FISHER INFORMATION

In this section, we discuss the QFI and the maximal mean QFI. Generally, for an input state ρ_{in} under a linear rotation by an angle φ , the output state can be written as $\rho_{\varphi} = e^{i\varphi J_{\vec{n}}} \rho_{\text{in}} e^{-i\varphi J_{\vec{n}}}$. According to the Quantum Cramer-Rao theorem, the phase sensitivity φ has a lower bound limit [30, 31]

$$\Delta\hat{\varphi} \geq \Delta\varphi_{\text{QCR}} = \frac{1}{\sqrt{vF(\rho_{\text{in}}, J_{\vec{n}})}}, \quad (1)$$

where $\hat{\varphi}$ is an unbiased estimator (i.e., $\hat{\varphi} = \varphi$), v is the number of experiments, and $F(\rho_{\text{in}}, J_{\vec{n}})$ denote the QFI which is defined as [30, 31]

$$F(\rho_{\text{in}}, J_{\vec{n}}) = \text{Tr}(\rho_{\varphi} L_{\varphi}^2). \quad (2)$$

Here, L_{φ} is the so-called symmetric logarithmic derivative determined by the following equation

$$\frac{\partial \rho_{\varphi}}{\partial \varphi} = \frac{1}{2} (\rho_{\varphi} L_{\varphi} + L_{\varphi} \rho_{\varphi}). \quad (3)$$

In Eq. (1), one notices that, besides increasing the number of experimental times v , we can improve the estima-

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tion precision $\Delta\hat{\varphi}$ by choosing a proper state for a given $J_{\vec{n}}$, which maximize the value of the QFI.

Now, we consider the maximal QFI for a given state. Based on Eq. (3), the expression of Eq. (2) is explicitly derived as

$$F(\rho_{\text{in}}, J_{\vec{n}}) = 2 \sum_{i \neq j} \frac{(p_i - p_j)^2}{p_i + p_j} |\langle i | J_{\vec{n}} | j \rangle|^2, \quad (4)$$

where $\{|i\rangle\}$ are the eigenstates of ρ_{φ} with eigenvalues $\{p_i\}$. Then the Eq. (4) can be compactly rewritten as

$$F(\rho_{\text{in}}, J_{\vec{n}}) = \vec{n} \mathbf{C} \vec{n}^T, \quad (5)$$

where the matrix element for the symmetric matrix \mathbf{C} is

$$\mathbf{C}_{kl} = \sum_{i \neq j} \frac{(p_i - p_j)^2}{p_i + p_j} [\langle i | J_k | j \rangle \langle j | J_l | i \rangle + \langle i | J_l | j \rangle \langle j | J_k | i \rangle]. \quad (6)$$

It can be seen that the rotation along the \vec{n} direction affects the sensitivity of the state ρ . For a pure state, the QFI can be expressed as $F(\rho_{\text{in}}, J_{\vec{n}}) = 4(\Delta J_{\vec{n}})^2$ [32]. To obtain the maximal QFI, we rewritten the variance as

$$(\Delta J_{\vec{n}})^2 = \vec{n} O (O^T \mathbf{C} O) O^T \vec{n}^T = \vec{n}' \mathbf{C}_d \vec{n}'^T, \quad (7)$$

where O is an orthogonal matrix, \vec{n}' is a new direction defined as $\vec{n}' = \vec{n} O$, and \mathbf{C}_d is the diagonal form of \mathbf{C} ,

$$\mathbf{C}_d = O^T \mathbf{C} O = \text{diag}\{\lambda_1, \lambda_2, \lambda_3\}, \quad (8)$$

where the λ_i 's are the eigenvalues of \mathbf{C} . Now the maximal variance reads

$$\max(\Delta J_{\vec{n}})^2 = \max[\lambda_1(n'_1)^2 + \lambda_2(n'_2)^2 + \lambda_3(n'_3)^2]. \quad (9)$$

In the above equation, the rotated direction is normalized and satisfies the condition $n_1'^2 + n_2'^2 + n_3'^2 = 1$. If we set $\lambda_{\text{max}} = \lambda_1$ as the maximal one of the eigenvalues, then $\vec{n}' = (1, 0, 0)$, and the original direction $\vec{n} = \vec{n}' O^T$.

Now we get the maximal QFI

$$F_{\text{max}} = 4\lambda_{\text{max}}. \quad (10)$$

For simplicity, we study the maximal mean QFI

$$\bar{F}_{\text{max}} = \frac{F_{\text{max}}}{N}, \quad (11)$$

where N is the number of atoms.

III. MODEL AND A CLASSICAL ANALOGUE

System of BECs trapped in a symmetric double well have been well studied in theories and experiments [33–36]. Due to the interaction between two wells, the system presents JO and nonlinear ST phenomena [18–20]. For the two weakly coupled BECs system, the Hamiltonian can be described as [37–39]

$$H = \Omega J_x + 2\kappa J_z^2, \quad (12)$$

where the angular momentum operators are defined in terms of the creation and annihilation boson operators $\hat{a}_{1,2}^\dagger, \hat{a}_{1,2}$ as

$$J_x = \frac{\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1}{2}, \quad (13)$$

$$J_y = \frac{\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1}{2i}, \quad (14)$$

$$J_z = \frac{\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2}{2}, \quad (15)$$

which obey the SU(2) Lie algebra. The parameter Ω describes the coupling between two wells, and κ denotes the effective interaction of atoms. In present work, we focus on the interaction strength $\kappa > 0$ and $\Omega > 0$. In the system, the total particle number $N = \hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2$ is a conserved quantity.

To obtain the classical dynamics approach, we use a generalized SCS as an initial state, which is defined formally as [40–42]

$$\begin{aligned} |\theta, \phi\rangle &= e^{-i\theta(J_x \sin \phi - J_y \cos \phi)} |j, -j\rangle \\ &= \sum_{m=-j}^j \binom{2j}{j+m}^{1/2} \frac{\tau^{m+j}}{(1+|\tau|^2)^j} |j, m\rangle, \end{aligned} \quad (16)$$

where j is the angular momentum quantum number, $j = N/2$, $\tau = e^{-i\phi} \tan \frac{\theta}{2}$, and $\binom{2j}{j+m}$ is the binomial coefficients. Under this SCS, the expectation values of the angular momenta are given by

$$\langle \theta, \phi | \vec{J} | \theta, \phi \rangle = \frac{N}{2} (\sin \theta \cos \phi, \sin \theta \sin \phi, -\cos \theta). \quad (17)$$

Meanwhile, we obtain the rescaled Hamiltonian (with constant terms dropped)

$$\begin{aligned} \mathcal{H} &\equiv \langle \theta, \phi | \hat{H} | \theta, \phi \rangle / j \\ &= \Omega \sin \theta \cos \phi + \kappa_r \cos^2 \theta, \end{aligned} \quad (18)$$

where $\kappa_r = (N-1)\kappa$. With the help of path integral in the representation of SCS, we get the Lagrangian \mathcal{L} for the system [43]

$$\mathcal{L} = N/2 [\hbar(1 - \cos \theta) \dot{\phi} - \mathcal{H}], \quad (19)$$

which is associated with canonical coordinate ϕ and canonical momentum $p_\phi = \hbar(1 - \cos \theta)$ [44]. For simplicity, we use $p_\phi = -\hbar \cos \theta$ as the canonical momentum. By setting $\hbar = 1$, the Hamiltonian can be rewritten

$$\mathcal{H} = \Omega \sqrt{(1 - p_\phi^2)} \cos \phi + \kappa_r p_\phi^2. \quad (20)$$

Then we obtain the following canonical Hamiltonian's equations of motion for p_ϕ and ϕ in the phase space

$$\dot{p}_\phi = \Omega \sqrt{(1 - p_\phi^2)} \sin \phi, \quad (21)$$

$$\dot{\phi} = 2\kappa_r p_\phi - \frac{\Omega p_\phi \cos \phi}{\sqrt{1 - p_\phi^2}}. \quad (22)$$

TABLE I: Fix points and stable regimes

Parameters	a	b	c	d
$\Omega > 2\kappa_r$	$\theta = \pi/2, \phi = 0$ stable	$\theta = \pi/2, \phi = \pi$ stable	N/A	N/A
$\Omega < 2\kappa_r$	$\theta = \pi/2, \phi = 0$ unstable	$\theta = \pi/2, \phi = \pi$ stable	$\theta = \arcsin[\Omega/(2\kappa_r)], \phi = 0$ stable	$\theta = \pi - \arcsin[\Omega/(2\kappa_r)], \phi = 0$ stable

It is noted that the motion governed by the above equations are similar to that described by the mean field approximation [18, 19].

The stationary state solution of equations (21) and (22) can be obtained by assuming $\dot{p}_\phi = \dot{\phi} = 0$. For the case of $\dot{p}_\phi = 0$, we get $p_\phi = 1, \phi = 0$, and $\phi = \pi$. For $\dot{\phi} = 0$, we obtain $p_\phi = 0, p_\phi = \sqrt{1 - (\frac{\Omega}{2\kappa_r} \cos \phi)^2}, p_\phi = -\sqrt{1 - (\frac{\Omega}{2\kappa_r} \cos \phi)^2}$. Under these conditions, we obtain several fixed points and list them in Table I. It clearly shows that for stronger interaction, i.e. $\Omega > 2\kappa_r$, in the phase space, there are two fixed points, corresponding to $\theta = \pi/2, \phi = 0$ and $\theta = \pi/2, \phi = \pi$, respectively. For weaker interaction $\Omega < 2\kappa_r$, two more stable fixed points appear which correspond to $\theta = \arcsin \sqrt{1 - (\frac{\Omega}{2\kappa_r})^2}, \phi = 0$ and $\theta = \pi - \arcsin \sqrt{1 - (\frac{\Omega}{2\kappa_r})^2}, \phi = 0$, respectively.

A. Dynamical stability

In order to find out the fixed-point bifurcations, we need to analyze the stabilities of the fixed points. We first discuss the two fixed points ($\theta = \pi/2, \phi = 0, \pi$). These points are interesting because they depend on neither tunneling strength nor self-collision interaction strength. We shall adopt the linear stability analysis that has wide applications in variable nonlinear systems. To begin with, we assume $p_\phi = p_\phi^0 + \delta p_\phi$ and $\phi = \phi^0 + \delta \phi$, where (p_ϕ^0, ϕ^0) denote one of the fixed points in the phase space, δp_ϕ and $\delta \phi$ represent the deviations in the population difference and relative phase from the fixed points, respectively. With Eqs. (21) and (22), we obtain the linearized equation

$$\frac{\partial}{\partial t} \begin{pmatrix} \delta p_\phi \\ \delta \phi \end{pmatrix} = M \begin{pmatrix} \delta p_\phi \\ \delta \phi \end{pmatrix}, \quad (23)$$

where M is the Jacobian matrix

$$M = \begin{pmatrix} -\frac{\partial^2 H}{\partial p_\phi \partial \phi} & -\frac{\partial^2 H}{\partial \phi^2} \\ \frac{\partial^2 H}{\partial p_\phi^2} & \frac{\partial^2 H}{\partial p_\phi \partial \phi} \end{pmatrix}. \quad (24)$$

The eigenvalues of the linearized equation may be real, pure or complex imaginary. As is well known, the eigenvalues of the Jacobian matrix depict the types and stabilities of the fixed points. By substituting $\theta = \pi/2$,

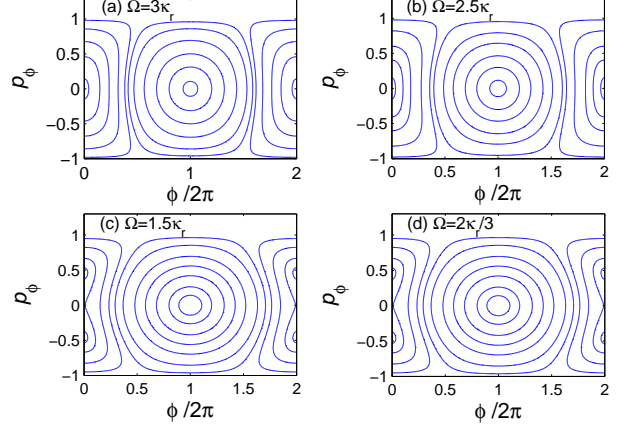


FIG. 1: (Color online) Trajectories of the Hamiltonian system in the phase space for various parameters. p_ϕ corresponds to the double well population difference and ϕ represents the phase difference between the two wells.

$\phi = 0, \pi$ into Eq. (24) and calculating the eigenvalues, we can find that the fixed point $\theta = \pi/2, \phi = 0$ is stable for $\Omega > 2\kappa_r$. When $\Omega < 2\kappa_r$, it becomes unstable. While for the fixed point $\theta = \pi/2, \phi = \pi$, it is always stable. Company to the condition that two new fixed points emerge for $\Omega < 2\kappa_r$, the classical bifurcation condition then can be obtained as

$$\Omega = 2\kappa_r. \quad (25)$$

The stabilities of the fixed points can also be seen from the trajectories of the Hamiltonian. In Fig.(1), we plot the evolution trajectories in the plane of θ and ϕ for various tunneling strengths. It shows that a fixed point is stable if the evolution trajectories are loops around a fixed point; otherwise it is unstable. Note that all the stabilities of the fixed points which obtained by the numerical simulation are consistent with the above theoretical analysis.

B. Dynamical transition between JO and ST regimes

Now we consider the transition from JO to ST regime. In the JO regime, the population difference oscillates symmetrically between two wells, and its average is zero. In the ST regime, the average of the population difference is nonzero, which can be obtained by the following

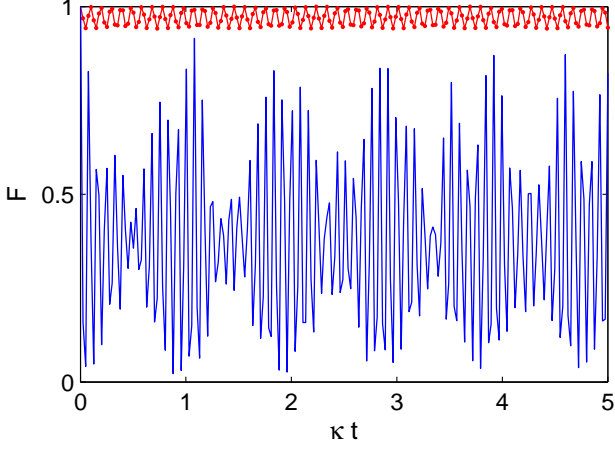


FIG. 2: (Color online) Fidelity of the overlap for different interaction of the two-mode BEC. The blue curve represents a weaker interaction with $\Omega = \kappa_r$, whereas the red curve with dots represents a stronger interaction with $\Omega = 4\kappa_r$. For both cases the initial state of the system is the SCS $|\theta = \frac{\pi}{2}, \phi = 0\rangle$ and the number of atom is $N = 100$.

condition [19]

$$\Omega \sin \theta_0 \cos \phi_0 + \kappa_r \cos^2 \theta_0 > \Omega \quad (26)$$

with θ_0 and ϕ_0 being the initial condition. From Eq. (26), we get the critical point of the transition

$$\Omega_c = \frac{\kappa_r \cos^2 \theta_0}{1 - \sin \theta_0 \cos \phi_0}. \quad (27)$$

For the initial value $\theta_0 = 0$, $\phi_0 = 0$, the transition parameter is $\Omega_c = \kappa_r$ which is consistent with the result of the model obtained by the mean field approximation [18, 24]. For $\theta_0 = \pi/6$, $\phi_0 = 0$, $\Omega_c = \frac{3\kappa_r}{2}$. If $\phi_0 = \pi$ and $\theta_0 = \pi/6$, the corresponding critical value becomes $\Omega_c = \frac{\kappa_r}{2}$. It shows that the critical point can be adjusted by the relative phase between two wells, and the relative phase can be experimentally adjusted by using a “phase-imprinting” method [45].

We emphasize that the particle number is large in the above classical analysis, however, in the practical experiment, the number of particles is finite. In the following discussions, we consider the quantum dynamic of the QFI to investigate the dynamical stability of the fixed point and the transition from JO to ST. We will consider two cases that the initial SCSs are chosen as $|\theta = \pi/2, \phi = 0\rangle$ and $|\theta = 0, \phi = 0\rangle$. The SCS $|\theta = 0, \phi = 0\rangle$ is simply a Dicke state $|j, -j\rangle$ in which all atoms lie in one of the wells. $|\theta = \pi/2, \phi = 0\rangle$ is a state with a well-defined phase difference $\phi = 0$ and is a phase state of a two-mode boson system.

IV. EFFECT OF STABILITY ON QUANTUM DYNAMIC

First we investigate the effect of stability on quantum dynamic of the system. We start with the initial state as $\theta = \pi/2$, $\phi = 0$, (i.e. $|j, j\rangle_x$) which corresponds to a fixed point. Such a state can be realized by applying a two photon $\pi/2$ pulse to the state $|j, j\rangle$ with all the atoms in the internal state $|F = 1, m_F = 1\rangle$ [46–50]. For this initial state, in classical analogue, it will change its stability at the classical bifurcation.

A. Effect of stability on Fidelity

In treating the quantum dynamical problem, it is helpful to bear in mind some results from quantum information theory concerning fidelity. Fidelity has been widely studied for the problems of transition probability in quantum mechanics. Mathematically, for two pure states, the fidelity is defined as

$$F = |\langle \Psi | \Phi \rangle|. \quad (28)$$

For the system with the initial wave function $|j, j\rangle_x$, the state at arbitrary time t can be expanded as [52]

$$|\Psi(t)\rangle = \sum_m c_m(t) |j, m\rangle, \quad (29)$$

and the amplitudes $c_m(t)$ obeys

$$\begin{aligned} i\dot{c}_m(t) = & 2\kappa m^2 c_m(t) + \frac{\Omega}{2} c_{m-1}(t) \sqrt{(j+m)(j-m+1)} \\ & + \frac{\Omega}{2} c_{m+1}(t) \sqrt{(j-m)(j+m+1)}. \end{aligned} \quad (30)$$

With the help of Eq.(16), the amplitude for the initial SCS can be calculated as

$$c_m(0) = \frac{1}{2^j} \binom{2j}{m+j}^{1/2}. \quad (31)$$

According to the Heisenberg function, we get

$$\dot{J}_x = -2\kappa(J_y J_z + J_z J_y), \quad (32)$$

$$\dot{J}_y = 2\kappa(J_x J_z + J_z J_x) - 2\Omega J_z, \quad (33)$$

$$\dot{J}_z = \Omega J_y. \quad (34)$$

Unfortunately, for this model, it cannot be solved exactly for a many-particle case. Numerical results of the wave function overlap between the initial and the evolved state $F = |\langle \pi/2, 0 | \Psi(t) \rangle|$ are plotted in Fig. (2). The red curve with dots correspond to $\Omega = 4\kappa_r$ and blue one correspond to $\Omega = \kappa_r$. For $\Omega = 4\kappa_r$, it can be seen that the fidelity oscillates around the value 1. It indicates that the dynamic does not take the state far away from the initial state. According to the stability analysis in Sec. III, the fixed point which corresponds to the initial state is stable

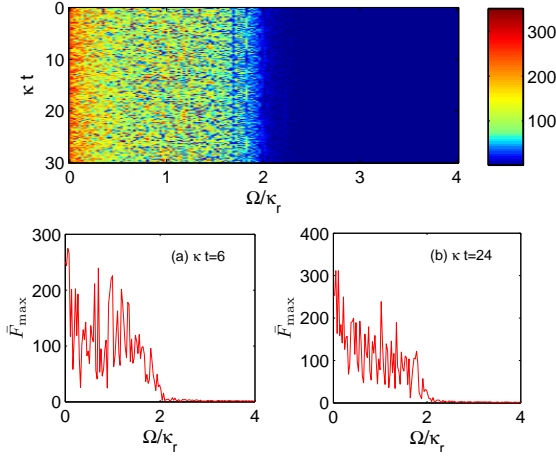


FIG. 3: (Color online) Top panel shows interaction-time plot of maximal mean QFI \bar{F}_{\max} . The bottom shows the \bar{F}_{\max} as a function of Ω/κ_r with rescaled time (a) $\kappa t = 6$ and (b) $\kappa t = 24$. Initial state of the system is SCS $|\theta = \frac{\pi}{2}, \phi = 0\rangle$ and the number of atom is $N = 500$.

in the regime $\Omega > 2\kappa_r$. When $\Omega = \kappa_r$, we can find that the fidelity oscillates between 0 and 1, and the amplitude of the oscillations is inhomogeneous. In the classical analogue, the fixed point is unstable in the regime $\Omega < 2\kappa_r$. From the above analysis, we find that there is a perfect classical-quantum correspondence.

B. Effect of stability on QFI

Now, we consider the effect of stability on the QFI. Numerical results of the maximal mean QFI \bar{F}_{\max} as a function of $\frac{\Omega}{2\kappa_r}$ and κt are plotted in Fig. (3). Obviously, the dynamic of the quantum system can be well illustrated by the maximal mean QFI. As is shown in the top panel of Fig. (3), the behaviors of the maximal mean QFI is quite different on the two sides of the classical bifurcation point $\Omega = 2\kappa_r$. To clearly depict the phenomenon, at the bottom of Fig. (3), we plot \bar{F}_{\max} as a function of Ω/κ_r for two arbitrary rescaled time $\kappa t = 6$ and $\kappa t = 24$, respectively. These two figures show that \bar{F}_{\max} behaves irregular oscillations in the regime $\Omega < 2\kappa_r$, in which the fixed point is unstable. When $\Omega > 2\kappa_r$, the fixed point is stable and we can find \bar{F}_{\max} oscillates with time around the initial value. Moreover, we can find that the value of the maximal mean QFI in the stable regime is much smaller than that in the unstable regime. The phenomenon can be understood by the fidelity which have been discussed in the above. In the stable regime, the evolved state is close to the initial SCS, and the \bar{F}_{\max} is small. While in the unstable regime, the evolved state far away from the initial SCS, and the \bar{F}_{\max} become bigger. These results suggest that the QFI can be well used to characterize the stability of the fixed point in this model.

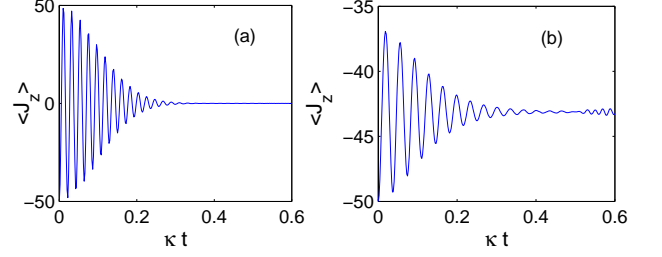


FIG. 4: (Color online) Quantum mechanical time evolution of $\langle J_z \rangle$ in the ST and JO regimes for the initial condition $|j, -j\rangle$, corresponding to all atoms initially in one of the wells. The plots in terms of a dimensional parameter κt , with (a) $\Omega = 3\kappa_r$; (b) $\Omega = 2\kappa_r/3$. The number of atom is $N = 100$.

V. DYNAMICAL TRANSITION BETWEEN JO AND ST REGIMES

A. population difference in different regimes

Below, we consider another dynamical problem, the transition from JO to ST regime. Firstly, we investigate the population difference between two wells. We choose $|\theta = 0, \phi = 0\rangle$ as the initial state, which corresponds to all atoms localized in one well. In this case, it is clearly that for $\Omega = 3\kappa_r$ the initial state is related to the JO regime, whereas for $\Omega = 2\kappa_r/3$ it is related to the ST regime. In Fig. (4a) and Fig. (4b), we plot the quantum evolution of $\langle J_z \rangle$ as a function of κt for the JO and the ST regimes, respectively. In the JO regime, $\langle J_z \rangle$ oscillates around zero during the evolution. It indicates that there is no preferential tunneling to any of the wells. While in the ST regime, $\langle J_z \rangle$ oscillates around a non-zero value and only in the half plane. It shows that part of the condensate is trapped in one of the wells. From the above results, we find that the dynamical properties of such a quantum system are quite different for the JO and the ST regimes.

B. QFI in different regimes

To well understand the quantum dynamical transition from JO to ST phenomenon, we calculate the time evolution of the maximal mean QFI. For a pure Dicke state $|j, m\rangle$, the maximal QFI is obtained as

$$F_{\max} = 2(j^2 + j - m^2). \quad (35)$$

Then for the initial state $|j, -j\rangle$, the maximal mean QFI can be got as $\bar{F}_{\max} = 1$. Numerical results of the maximal mean QFI \bar{F}_{\max} with different interactions are plotted in Fig. (5). From the top of Fig. (5), it is clearly seen that the behavior of the maximal mean QFI are quite different in the JO and the ST regimes. To well describe the behaviours of the maximal mean QFI for different regimes, we plot the maximal mean QFI as a function of

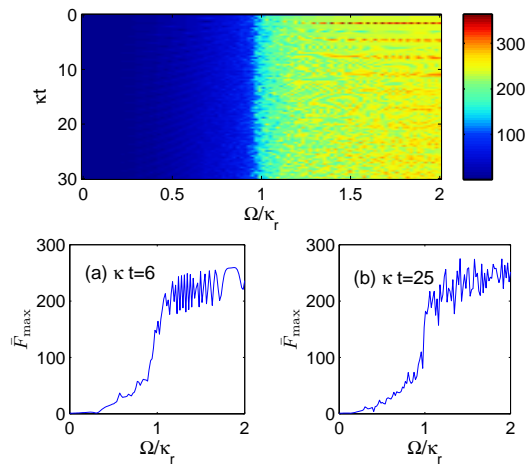


FIG. 5: (Color online) Top panel shows interaction-time plot of maximal mean QFI \bar{F}_{\max} . The bottom shows the \bar{F}_{\max} as a function of Ω/κ_r with rescaled time (a) $\kappa t = 6$ and (b) $\kappa t = 25$. Initial state of the system is the SCS $|\theta = 0, \phi = 0\rangle$ and the number of atom is $N = 500$.

Ω/κ_r with two rescaled time $\kappa t = 6$ and $\kappa t = 25$ in the bottom of Fig. (5). From these two figures, we can see that, for $\Omega < \kappa_r$, the maximal mean QFI increases slowly. As the interaction strength approach the transition point $\Omega_c = \kappa_r$, the \bar{F}_{\max} increases quickly. When $\Omega > \kappa_r$, it oscillates quickly and its value is much larger than that in JO regime. It is shown that the QFI can be served as an indicator of the transition from JO to ST regime. Such abrupt change of the QFI also can be found in the phase transition of spin systems.

In experiments, so far, many observers have found number squeezing in this model [53]. Moreover, some researchers have shown that the number fluctuation has

a nontrivial relation with the phase fluctuation, even for a single-mode light field [54]. We hope that the transition of the maximal mean QFI for this system can be observed in experiment in the future.

VI. CONCLUSION

In conclusion, we have studied the dynamical evolution of a two mode BEC in a symmetric double well. All the relevant fixed points and their stabilities were analyzed. We investigated the dynamical behavior of maximal mean QFI in this system which well described the stability of the fixed point. For the initial state $|\theta = \pi/2, \phi = 0\rangle$, the numerical result showed that \bar{F}_{\max} has a small oscillation around the initial value in the $\Omega > 2\kappa_r$ regime, while in the regime $\Omega < 2\kappa_r$, we found that \bar{F}_{\max} displays a strong irregular oscillation.

We also investigated the maximal mean QFI in the JO and the ST regimes which corresponds to the different distribution of particles in two wells. For the initial state $|\theta = 0, \phi = 0\rangle$, we found the mean maximal QFI increases quickly at the critical point which corresponds to the boundary between the JO and the ST regimes. From these results, it showed that the QFI not only characterizes the stability of the fixed point, but can also signal the presence the transition from JO to ST regime.

VII. ACKNOWLEDGMENTS

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